TRANSIENT TEMPERATURE DISTRIBUTIONS IN A CYLINDER HEATED BY MICROWAVES

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ABSTRACT

Transient temperature distributions were calculated for a lossy dielectric cylinder coaxially aligned in a cylindrical microwave cavity excited in a single mode. Results were obtained for sample sizes that range from fibers to large cylinders. Realistic values for temperature dependent complex dielectric constants and thermophysical properties of the samples were used. Losses in cavity walls were taken into account as were realistic thermal emissivities at all surfaces. For a fine mesh of points in time, normal mode properties and microwave power absorption profiles were evaluated using analytic expressions. Those expressions correspond to exact solutions of Maxwell's equations within the framework of a cylindrical shell model. heating produced by the microwave absorption was included in self-consistent numerical solutions of thermal equations. In this model, both direct microwave heating and radiant heating of the sample (hybrid heating) were also studied by including a lossy dielectric tube surrounding, the sample. implications of the calculated results for materials processing will be discussed.

INTRODUCTION

Time dependent temperature behavior of a cylinder heated by microwaves was calculated with the aid of a realistic model of a single mode cylindrical cavity for problems of interest in materials processing, The model has been described in previous articles [1,2] where it was applied to calculate the microwave absorption and steady state temperature profiles [2] for a cylindrical sample. This new treatment extends those cylindrical sample studies along with other earlier work concerned with spherical samples in single mode cavities [3 - 5].

In this work, as well as our earlier studies, electromagnetic properties of a cavity partly filled with a sample are treated analytically using a shell model. The results provide formulas for evaluating microwave power absorption per unit volume, That absorption is included as a heat source in thermal equations that are solved using a combination of analytic and finite difference methods while taking into account the temperature dependence of thermophysical properties of the sample [6]. So far our models have not included heat of chemical reactions and changes of material properties as the sample is processed and undergoes densification, changes in chemical composition, or changes in phase. However, we plan to include some of those effects in advanced models in the future.

The present treatment of a cylindrical shell model of a microwave processing reactor has several noteworthy features. First, Maxwell's equations are solved exactly and the electrical conductivity of cavity walls is taken into account accurately. Second, realistic values of thermal emissivity are taken into account at solid boundaries inside the cavity and at the cavity walls. Third, the model can take into account direct microwave heating of the sample and hybrid heating, which additionally involves radiant heating of the sample by a microwave-heated tube that surrounds it.

A variety of applications can benefit from calculated time-dependent temperature profiles. Two important examples are the prediction and control of preheating and processing conditions, and design and optimization of new reactors. in this article, the usefulness of the calculated results will be illustrated first by focusing on parametric studies that search for evidence of thermal runaway during materials processing. A second illustration is concerned with investigation of the relative merits of processing fibers and large samples using microwave heating alone or hybrid heating. The theory is described next. Then calculated results are presented and discussed. Our conclusions are presented in the final section.

THEORY

The theory is based on a shell model represented in Fig. 1. The sample is a lossy dielectric rod aligned along the axis of the cavity. A lossy dielectric tube surrounding the rod is included when hybrid heating of the sample is studied, Otherwise the tube is absent and the rod is heated by microwaves alone. l-lest transfer occurs by thermal conduction alone inside the material of the rod and tube. Heat transfer by thermal radiation occurs in all vacuum spaces. This radiation is treated in a gray body approximation with realistic thermal emi ssivities at all curved solid boundaries in the cavity. The flat end plates are treated as perfect reflectors for thermal radiation [7]. In practice this can be approximated by highly polished copper end plates. There is no thermal path from rod or tube to the end plates because the path is interrupted by a very small gap at each end. This model then permits the thermal radiation problem to be treated analytically, for it is as if the cylindrical surfaces were infinitely long[8].

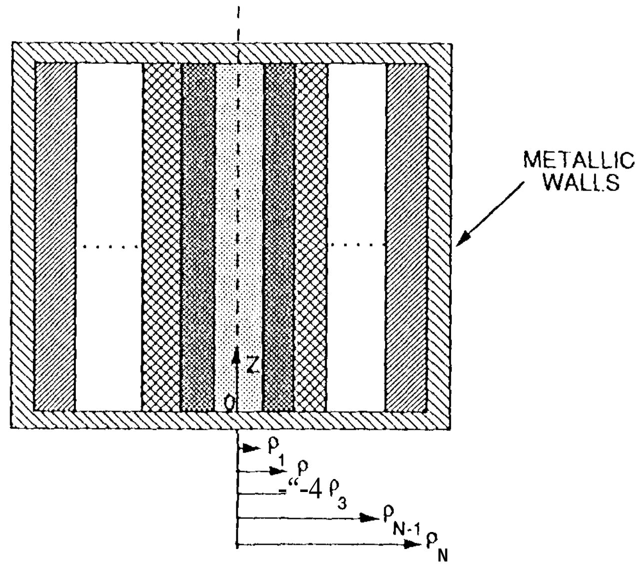

Fig. 1. Geometry of cylindrical rod and cavity.

The sample and the tube are partitioned into many thin cylindrical zones or shells, however the vacuum spaces are not subdivided. The curved cavity wall is treated as one of the zones. The complex dielectric constant is taken to be uniform in each zone, but it may vary from one zone to another, thus allowing for the temperature dependence of dielectric constant in the rod and t u be. The cavity walls are taken to be at some uniform temperature. Maxwell's equations can be solved exactly for this shell model, where boundary condi t ions are matched at interfaces between shells and assuming perfectly electrically conducting end plates. The normal mode properties of this system of shells can be found for any assigned distribution of complex dielectric constant using a known 4 x 4 matrix formalism [9]. Any normal mode can be treated analytically by that technique, but here we will focus on TMono mocks, where there is no angular dependence or z-dependence in the. fields. For these modes, microwave power absorption per unit volume varies spatially only in the radial direction. That absorbed power distribution varies as the temperature profile in the cavity reactor changes with time. Microwave absorption by the cavity end plates is calculated using a surface resistance approximation.

The solution of the electromagnetic problem was combined with thermal equations to calculate temperature distributions with microwave absorbed power density as a heat source. Under transient conditions, the general forms of the thermal equations inside the sample or tube are

$$q(r,t) = -\kappa(r,t)\,\nabla T(r,t) \tag{1}$$

$$\nabla \bullet q(r,t) = P(r,t) - C(r,t)\dot{T}(r,t)\,. \tag{2}$$

$C(r,t)$ is the heat capacity per unit volume and all other symbols have their usual meanings. Heat transfer through vacuum by thermal radiation between curved solid boundaries, say 1 and 2, can be evaluated exactly in a gray body context for this model using the following formula for net radiation exchange, Q, between two infinitely long cylinders:

$$Q = \frac{\sigma_{SB} A_1 \left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{\rho_1}{\rho_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)} . \tag{3}$$

The index 1 refers to the inner surface, where A_1, T_1, ε_1, and ρ_1 are respectively surface area, temperature, emissivity and radius. A combination of analytic and finite difference methods applied to the shell model provides an efficient means for treating the thermal problem self-consistently with the electromagnetic equations. This involves a two-step procedure, applied alternately for as many repetitions as desired, to propagate the system forward in time. First, for an assigned set of complex dielectric constants in the shells, the electromagnetic problem is solved and a microwave absorbed power density distribution is calculated. Then the thermal equations are used to calculate an updated temperature profile for a slightly later time. This is used to update the complex dielectric constant distribution and the procedure is repeated. Some results calculated with this self-consistent procedure will be presented and discussed next.

DISCUSSION

This transient model is an extension of our previous models for calculating the absorption [1] and steady state temperature profile [2] for a cylindrical rod aligned along the axis of a cylindrical microwave cavity. The present calculations correspond to the same nominal set of experimental parameters used previously [2], unless stated otherwise. The cylindrical cavity had a radius ρ_c = 4.69 cm and a length L = 6.63 cm corresponding to an empty cavity TM_{010} mode resonant frequency of 2.45 GHz. The nominal alumina rod had radius and emissivity values of a = 0.2 cm and ε_s = 0.31 respectively. The cylindrical cavity walls were copper with an emissivity of ε_w = 0.025, Hybrid heating studies were performed using an alumina tube surrounding the rod that had a thickness, mid-radius and inner and outer surface emissivities of d = 0.1 cm, r_{mid} = 0.4 cm and $\varepsilon_{t_i} = \varepsilon_{t_o}$ = 0.31 respectively. The temperature dependence of the real and imaginary dielectric constant for a 99.8% pure alumina rod [10] was used in these calculations. A Y-MP2E Cray computer performed the calculations using 8 zones for the rod and 2 to 8 zones for the tube.

The time-temperature heating response for several power levels, using the nominal set of parameters without the presence of a surrounding tube, is shown in Fig. 2. Results are shown for the center and surface of the 0.2 cm radius alumina rod. The time required to reach steady state decreases with increasing power even though the steady state temperature is higher. For a power level of 150 watts, the center temperature of the rod was 1756°C while the surface temperature was only 46°C lower. This small temperature gradient is in sharp contrast to large temperature gradients previously obtained for spherical samples [5].

The addition of an alumina tube around the rod leads to hybrid heating effects. Figure 3 shows the temperature profiles within the rod and tube at various times during heating with 100 watts of input power. After 300 seconds, the rod and tube have essentially reached steady state profiles. At steady state, the 100 watts is distributed between the cavity walls (0.4 watts), the tube (41.9 watts) and the rod (57.7 watts). The rod center and surface temperatures reach 1611 .3°C and 1595.2°C respectively. This temperature gradient of 16.1 °C is almost half the 26.8°C gradient obtained with no tube from the 100 watt data in Fig. 2. This reduced gradient is caused by the radiant heating of the rod surface due to the surrounding tube, The hybrid heating also leads to a higher rod steady state temperature of= 67°C. The tube inner and outer surfaces reach steady state temperatures of 1335.4°C and 1325.5°C respectively after 300 seconds which is ≈270°C lower than the rod.

The calculations from this new transient model were compared to the results obtained from the previous steady state model [2]. The near steady state 300 second profile in Fig. 3 was less than 0.2% below the steady state model results for both rod and tube temperatures. This excellent agreement validates the accuracy attainable with the transient model.

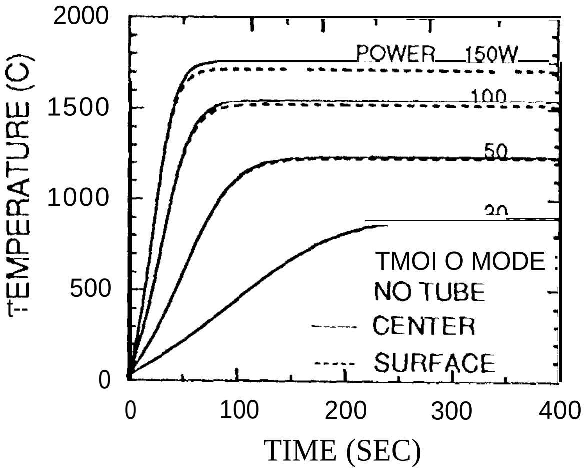

Fig. 2. Transient response of a 0.2 cm radius alumina rod for various power levels.

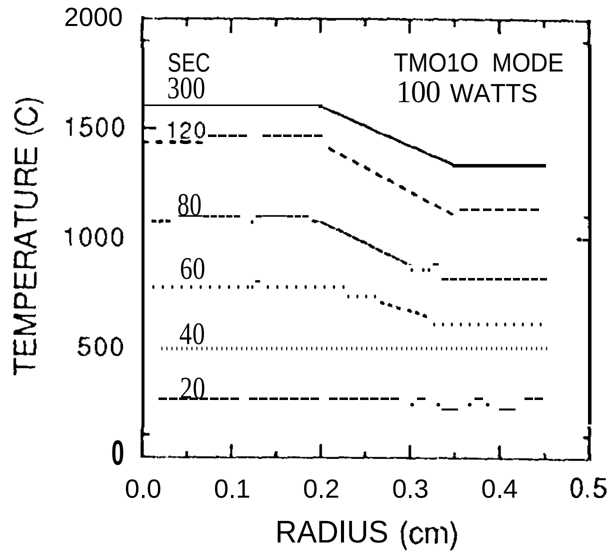

Fig. 3. Temperature profiles in rod and tube during heating with 100 watts.

In a previous study, the steady state temperatures at the center and surface of rods of various radii were calculated with and without a surrounding tube for a TM010 mode power level of 30 watts (see Fig. 7 of ref. 2). 'I'hat study found a maximum sample temperature of 1470°C with no tube (and 919°C with a tube) for a rod radius of 0.025 cm. Figure 4 shows the time-temperature transient behavior for the center of the 0.025 cm radius rod with and without the tube present for the same experimental conditions.

A parametric study was performed to compare the transient time require to reach steady state for various sample sizes ranging from fibers to large rods. Figure 5 shows the center sample temperature versus t ime for four rod radii. As expected, the smaller fiber-like samples were first to reached steady state (in ≈ 100 seconds) while the the large 2 cm radius rod took over 10^4 seconds. The

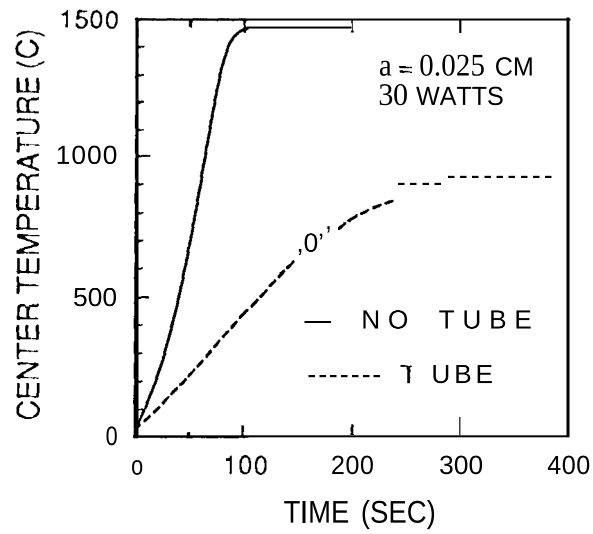

Fig. 4. Transient heating behavior for a 0.025 cm radius rod with and without a surrounding tube.

effects of hybrid heating, using the surrounding tube, on the transient response is shown in Fig. 6. The radiation contribution from the surrounding tube. has a significant effect on the small fiber-like samples. The transient response is essentially the same for the 0.0125 and 0.025 cm radius samples. This result is due to the fact that the tube radiation dominates the heating of these samples. The insertion of the tube raises the 0.0125 cm radius sample and lowers the 0,025 cm radius sample steady state temperatures with no tube present. 'I'he time to reach steady state conditions is also slightl y increased in the presence of the tube. In the case of the larger 0.2 and

2.0 cm radius rods the tube slight] y increases the **final** steady state temperatures, however it takes slightly longer to reach equilibrium conditions.

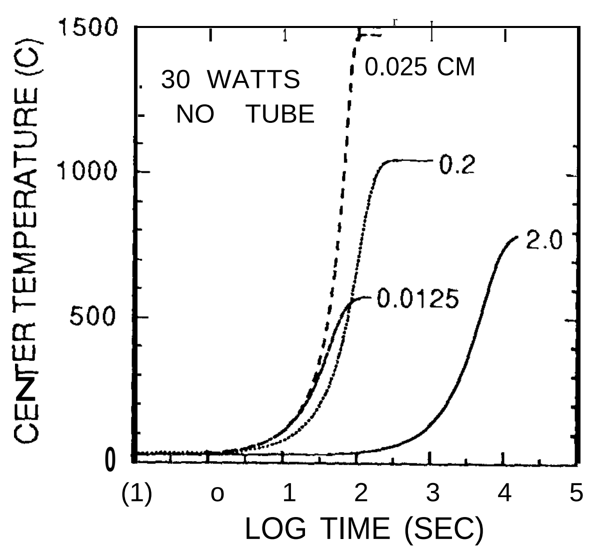

Fig. 5. Transient response of samples ranging from fibers to large rods with no surrounding tube.

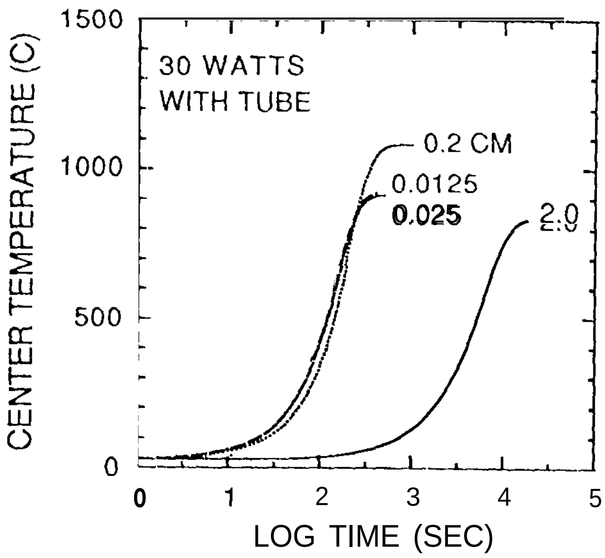

Fig. 6. Transient response of samples ranging from fibers to large rods including a surrounding tube.

THERMAL RUNAWAY

One of the main concerns regarding the application of microwave heating techniques to commercial applications is the possibility of thermal runaway. Thermal runaway can be interpreted as follows. For temperatures below some threshold, the sample attains steady state for a constant power input to the cavity. For an incremental power increase above the threshold the steady state condition is not attained and in some portion of the sample the temperature increases to the melting point. Under these conditions, the microwave power absorbed in some region of the sample becomes greater than the ability to transfer the heat away to the surrounding regions. The present transient model is ideally suited to test for thermal runaway in cylindrical samples.

To investigate thermal runaway, we first evaluated the transient response for rods of various radii using the experimentally determined complex dielectric constant for 99.5% pure alumina [10]. Figure 7 shows the transient response for three rods ranging over ≈ two orders of magnitude in radius using the. experimentally determined complex dielectric constant. A typical experimental wall emissivity of 0.3 was used for these calculations. The power level was continually increased for each rod in an attempt to obtain a steady state value near the melting point of alumina (≈ 2000°C). 'l'he present transient model assumes that the normal mode resonant frequency is continuously tracked during heating. The curves in Fig. 7 for the 0.025, 0.2, and 2.0 cm radii rods correspond to power levels of 55, 300, and 950 watts respectively. While the slope of the time-temperature curves are steep, there is no indication that thermal runaway occurs.

It has been speculate that thermal runaway is associated with an increase in the temperature dependence of the loss tangent at higher temperatures. To test this hypothesis, we defined two hypothethical expressions for the temperature dependence of the complex dielectric constant ε'' that are modifications of the experimentall y determined exponential expression. The hypothethical $\varepsilon''(T)$ expressions are given by

$$\varepsilon''(T) = Ae^{[T/B_1]} \tag{4}$$

$$\varepsilon''(T) = Ae^{\left[(T+C)/\tau\right]} \tag{5}$$

where A = 0.00438 and τ = 309.49.

The parameters B and C in Eqs. 4 and 5 can be used to amplify the exponential temperature dependence. The actual experimental exponential expression can be obtained by setting B = 1 in Eq. 4 or C = O in Eq. 5. The enhanced temperatur e dependence of the imaginary dielectric constant using Eqs. 4 and 5 is shown in Fig. 8. Values of the parameters B and C were chosen to cover the range O < < 100. A transient response investigation of thermal runaway using these was performed for B and C temperature dependence shown in Fig. 8. For all cases, as the input power was increased, steady state conditions were obtained all the way up to melting.

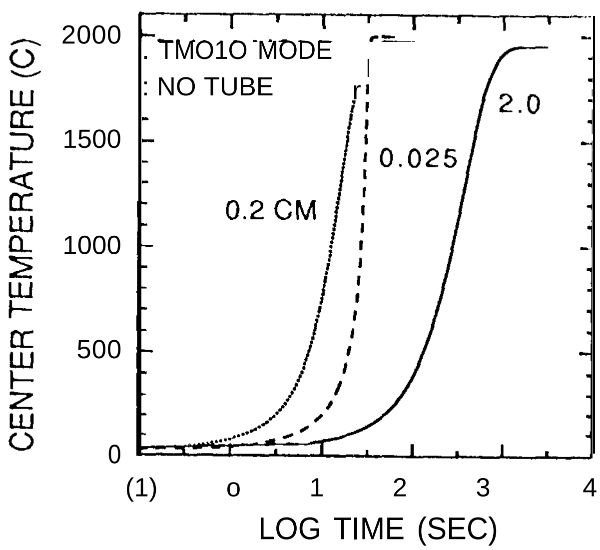

Fig. 7. Transient response for various rod radii using nominal temperature dependence of complex dielectric constant [1O].

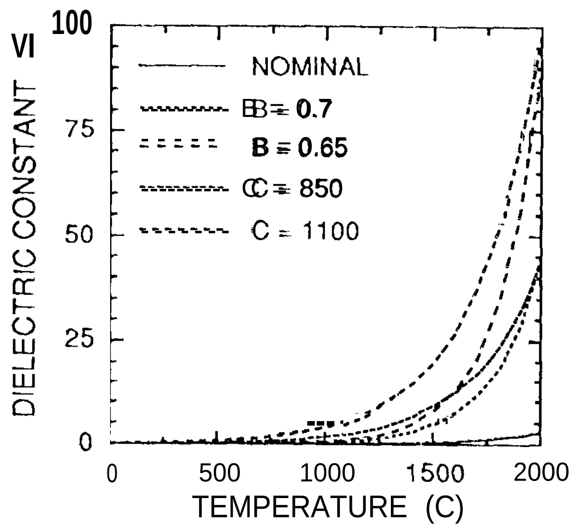

Fig. 8. Temperature dependence of the imaginary dielectric constant using the expressions in Eqs. 4 and 5.

CONCI.US1ONS

The results of the present study suggest that there is no thermal runaway phenomenon associated with the microwave heating of a cylindrical rod aligned along the axis of a cylindrical cavity. How can this conclusion be'reconciled with experiment reports of thermal runaway in microwave heated rods? A possible explanation is based on three important facts: (1) the slope of the temperature versus power curves can become quite steep, (2) the excitation mode resonant frequency is not being tracked in must experimental situations and (3) it can take considerable time to reach steady state conditions. In practical experimental situations, the microwave power is usually continually increased until a desired temperature is reached. If researchers are not patient, the power may be increased too quickly to a level that causes the sample to inadvertently reach the melting temperature. This senario should be tested using controlled experimental st udies.

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